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# The Productivity Gaps of Female-Owned Firms: Evidence from Ethiopian Census Data

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## Abstract

This paper provides new empirical evidence on the relative productivity disadvantage of female-owned firms compared to male-owned firms in a developing country setting. We rely on a large panel of Ethiopian manufacturing firms based on an annual census run by the Central Statistical Agency of Ethiopia over 2003-2009. Our preferred estimation shows a 12% difference in levels of total factor productivity between female- and male-owned firms. Moreover, drawing on novel quantile approaches to formally compare productivity distributions, we dig deeper into some of the potential mechanisms underlying this gender-based firm productivity gap.

## Keywords

Manufacturing firms, productivity distribution, gender gap, Ethiopia

## Introduction

Closing gender gaps has become an imperative in global development circles. While much progress has been recorded under the Millennium Development Goals, the recently established Sustainable Development Goals call for renewed efforts to reduce gender discrimination and increase empowerment, with a stronger emphasis on the promotion of a more active role for women as decision makers and owners of economic resources. As a matter of fact, in many developing countries, including in sub-Saharan Africa (SSA), issues relating to the extent and nature of women's participation in productive activities, to the allocation of women into specific sectors and types of jobs, and to high gender inequalities in the labour market remain pressing (Menon and Rodgers 2009; Hallward-Driemeier 2011; Juhn et al. 2014). Also, much is still unknown about which and how discriminatory practices and institutional constraints to which women are subjected influence their entrepreneurial capabilities.

The literature on gender, entrepreneurship, and firm performance generally shows evidence of significant gender gaps both at market entry (i.e., women are less likely to become entrepreneurs) as well as in several dimensions of female-owned firms' performance (Hellerstein and Neumark 1999; World Bank 2012; Marques 2015). A good part of this literature concerns industrialised rather than low-income countries, and micro- and small entrepreneurs rather than larger enterprises (see Klapper and Parker 2011 for a more complete overview). The evidence for transition and developing countries is mixed (Sabarwal and Terrell 2008; Bruhn 2009), and the few studies including SSA in their analysis generally do not observe stark gender-based firm performance differences (Bardasi et al. 2007; Bardasi et al. 2011). Conversely, SSA country-specific evidence suggests considerably lower female firm productivity in Ghana (Jones 2012) and Madagascar (Nordman and Vaillant 2014). The lack of systematic evidence may be due to the type and quality of the data used, as well as to the definition of female ownership adopted (Aterido and Hallward-Driemeier 2011).

Furthermore, there is no conclusive evidence on the underlying factors explaining any gender gaps in firm performance in developing countries and SSA more specifically, although several hypotheses have been advanced (see Campos and Gassier 2017 for a recent, comprehensive survey). In many countries, female businesses are concentrated in sectors characterised by limited economies of scale, low growth, low technology and capital, and intense competition (World Bank 2012; Kucera and Tejani 2014). Also, female entrepreneurs may be disadvantaged in terms of education, experience and other skill-related traits that are positively linked to productivity (Aterido and Hallward-Driemeier 2011). Yet, the empirical work by Nix et al. (2016) on microenterprises based in the Republic of Congo, Ghana, Rwanda, and Tanzania shows that the gender performance gap remains partly unexplained even after controlling for industry, firms, and owners' characteristics. Similar conclusions are reached by Hardy and Kagi (2018a), who look into the case of Ghana specifically. The role of adverse external conditions, including access to credit and capital, has been tested in experimental work, where it is shown that returns to capital grants tend to be positive for men but close to zero for women (de Mel et al. 2008, 2009; Fafchamps et al. 2014).[2] Finally, a recent study on Ghana by Hardy and Kagi (2018b) seems to suggest the existence of demand-side constraints, too. Limited formal employment opportunities for women increase the number of female micro-entrepreneurs, thereby generating lower market-size-to-firm ratios and higher demand-scarcity for female-owned firms.

In this paper, we aim at addressing some of above issues and complement the empirical literature on gender-based firm productivity differences. We rely on a large panel of Ethiopian manufacturing firms based on an annual census run by Ethiopia's Central Statistical Agency (CSA) over 2003-2009. The case of Ethiopian manufacturing is particularly interesting and relevant for our purposes. Over the last decades, the country has experienced sustained economic growth, spurred by large infrastructural investments, waves of trade liberalization, and industrial policies supporting the growth of the manufacturing sector and the structural transformation of the economy (World Bank 2015), aimed at fostering broader economic participation and enhancing productivity. Importantly, however, large gender gaps remain in the Ethiopian labour market. Female entrepreneurship is still limited in the country because of the existence of higher barriers for women than for men, including in access to finance and education (Alibhai et al. 2017). A few previous studies that have looked into gender-based heterogeneity in firm productivity in Ethiopia based on survey data have reported a negative effect of female business ownership on productivity, especially in rural areas (Bardasi and Getahun 2009; Rijkers and Costa 2012; Rijkers et al. 2010). The country performs relatively poorly on the World Economic Forum's Gender Gap Index (ranked 115$^{th}$ out of 144 countries in 2017), in part due to low scores on the 'economic participation and opportunity' sub index, which considers women's labour force participation, gender wage inequality and the prevalence of women in senior professional and technical positions (WEF 2017). A report by the ILO (2015) adds that only 22% of managers in Ethiopia are women, which puts it in the group of worst-performing countries in this specific area. That notwithstanding, more recent work has found that recent policy efforts, such as the combined reforms in the family code and in community-based land registration, have already brought measurable improvements in women's economic participation and welfare (Hallward-Driemeier and Gajigo 2015; Kumar and Quisumbing 2015).

The objective of our empirical analysis is twofold. First, along the lines of existing studies on the topic, we aim at understanding whether and to what extent female-owned firms in the Ethiopian manufacturing sector exhibit a performance gap compared to their male-owned counterparts. We

[2] Household capture may help explain this striking finding. Recent work by Bernhardt et al. (2017) using experimental data from India, Sri Lanka and Ghana shows that returns to capital in female-operated microenterprises receiving grants are lower in multiple enterprise households but not in single enterprise households, suggesting that women may be investing the capital in their husbands' enterprises rather than their own. In line with this, Fafchamps et al. (2014) find that female enterprises' profits in Ghana respond more positively to in-kind grants, which are less easily diverted to other purposes.

do so taking total factor productivity (TFP) as our measure of firm performance. Unlike the extant literature on other developing countries, including on Ethiopia, which mostly relies on survey data and includes large numbers of small and informal firms, we base our analysis on census data (thus avoiding problems of sample representativeness) and consider the universe of formal firms in the Ethiopian manufacturing sector.[3] Though this choice has the disadvantage of leaving out the majority of firms based in the country, which are small and informal, we are confident about the relevance of our analysis given that the firms in our sample represent roughly 50% and 80% of total employment and value added in the Ethiopian manufacturing sector, respectively.[4] To our knowledge, we are the first to employ firm census data from a SSA country for these purposes. The richness of our dataset allows us to check the robustness of female-male firm productivity differences to alternative definitions of female ownership, based either on capital shares or on the number of female owners, and to control for a large number of observable characteristics likely to affect the productivity gap.

In the second part of the paper we look beyond *average* gender-based firm productivity gaps and investigate differences between the productivity *distributions* of male- and female-owned firms. We do this in two ways. First, we apply the Oaxaca-Blinder-type decomposition approach designed by Firpo et al. (2007, 2009), which allows us to look at differences across male and female firms at different quantiles of the productivity distribution. This type of analysis enables us to identify how much of the productivity gap can be attributed to differences in observable firm characteristics/factors (what we will label as the composition effect) and how much by different returns to those factors (the structural effect) at various points of the distribution. Second, we draw on a quantile-based empirical strategy originally developed by Combes et al. (2012). Applied to our context, this quantile approach allows us to evaluate in which ways the shape of the firm productivity distribution varies along the gender of firm owners. More specifically, we formally distinguish between three 'transformations', i.e., shift, dilation and truncation, in the productivity distribution of female-owned firms relative to that of male-owned firms.

Our regression results show a consistent productivity gap for female-owned firms. The difference in productivity varies considerably, depending on the exact specification and definition of female ownership adopted (with stricter measures of female control over the firm resulting in larger estimated gaps). Our preferred estimation show that female-owned firms are about 12% less productive than male-owned firms, a result which is in line with previous, survey-based findings

[3] This marks another difference with existing studies looking at the gender productivity gap in Ethiopia (e.g. Rijkers et al. 2010; Rijkers and Costa 2012), given that they mostly focus on firms based in rural areas, whereas our firms are all based in urban areas.

[4] These numbers are computed comparing values of census data with the aggregate figures provided by the report based on the Ethiopian Small Scale Manufacturing Industry Survey (SSIS), a representative survey of manufacturing firms with less than 10 employees, for the latest year for which the two were run contemporaneously: 2008 (CSA 2010). For that year, the SSIS reports 43,338 small firms (53.2% of which are grain mills), engaging 139,000 people and generating 1.14 billion birr in value added. The 2008 census data, on the other hand, include 1,500 firms, engaging 134,000 people and producing around 5.9 billion birr of value added.

for large-scale, formal firms in other SSA countries (Aterido and Hallward-Driemeier 2011), but lower compared to other studies that include also smaller, informal and rural firms (Rijkers et al. 2010; Jones 2012; Nordman and Vaillant 2014). Our analysis further shows that lower capital intensity and smaller size; allocation into less productive, more labour-intensive and female worker-dominated industries are among the mechanisms explaining (part of) the female firm productivity gap.

Next, we provide new insights based on a formal comparison of female and male firm productivity distributions. Results of the Firpo et al. (2007, 2009) quantile decomposition analysis suggest that a consistent part of the productivity differences for the firms populating the middle section of the productivity distribution can be attributed to structural effects, and mainly to differences in the returns to capital. This echoes existing research based on field experiments with capital grants (de Mel et al. 2008, 2009). Conversely, productivity gaps in the upper part of the productivity distribution tend to be related to composition effects, i.e., differences in observed characteristics, and especially in firm size, capital endowments, and the internationalization of firms.

Results based on the Combes et al. (2012) approach show evidence of a significant leftward shift and lower dilation of the female firm productivity distribution compared to the male distribution. Interestingly, we also uncover that a (though very small) part of the female firm productivity gap can be explained by female firms' productivity distribution displaying a longer left tail. When we consider a sample confined to firms in the capital region, Addis Ababa, where more than half of all manufacturing establishments are located, the economic significance of the left shift and lower dilation is strengthened. Patterns become slightly more heterogeneous once we split the sample into smaller subgroup and when accounting for certain characteristics of the firms and of the sectors.

How to reconcile these various pieces of evidence? Taken together, our results indicate that female-owned firms are not only systematically less productive but, on the whole, also more homogeneous in terms of productivity than male-owned firms; this homogeneity is likely the result of female firms' concentration in certain, low-productivity sectors, such as the food processing industry. On the one hand, as shown by our decomposition analysis, the productivity differences in the central part of the distribution remain largely unexplained by observable characteristics of the firms. Rather, they seem to be driven by differences in returns to capital, whose origins and motivations have not yet been fully identified by the existing literature (Fafchamps et al. 2014; Bernhardt et al. 2017; Hardy and Kagi 2018b) and which, unfortunately, we cannot explore further with our data. On the other hand, we find that there is only a very small number of highly productive female firms that coexist with a much larger group of substantially less productive establishments. In the upper tail of the distribution, particular observable characteristics of female firms, such as relatively small size and limited capital seem to make it harder for them to stand out.

The remainder of the paper is structured as follows. The next section introduces our data and methodology. We then present the regression results for the average female-male firm productivity gap, and introduce and implements the two quantile approaches by Firpo et al. (2007, 2009) and Combes et al. (2012) to compare female and male firm productivity distributions.

## Data and descriptive analysis

This paper uses manufacturing firm-level data from Ethiopia over the period 2003-2009. Data come from the annual census of large and medium manufacturing firms run by the CSA, which covers all firms that engage more than ten people and that employ electricity in their production process.[5] All firms need to comply with CSA requirements, and the census therefore represents the universe of more structured, formal firms in the country.

This dataset, in various versions and guises, has been employed in the past to study firm growth, survival and structural change (Bigsten and Gebreeyesus 2007; Shiferaw 2009; Söderbom 2012); the role of exporting and trade liberalisation on productivity (Bigsten and Gebreeyesus 2009; Fiorini et al. 2018); returns to capital in formal vs. informal firms (Siba 2015); job creation, job destruction and skill-biased technological change (Shiferaw and Bedi 2013), and road infrastructure and firm entry (Shiferaw et al. 2015), among other topics. Importantly however, to our knowledge, it has never been used to evaluate gender-based firm productivity differences. We believe the dataset's extensive coverage of medium and large manufacturing enterprises and rich details on ownership structure make it particularly suitable for our purposes.

The dataset includes detailed information on output, capital, labour and other inputs for all firms. In addition, it provides precise information on the location of firms by their region, zone, district (*woreda*) and town of origin. It also contains data on the industry of firms' activities, up to the 4-digit level of the ISIC (revision 3) classification.

Crucially, the census includes very granular information on the ownership structure of each firm. Constructing a good gender-based measure of ownership from firm-level data is not trivial. In fact, the definition of what constitutes a 'female business' and the distinction between ownership and control matter a great deal in assessing productivity differences.[6] Most studies have used a generic measure of female participation in the ownership of a firm, i.e., a dummy that equals one if *any* woman is among the owners. While this perfectly fits the case of sole-owned firms, it may be a less precise measure of female ownership in firms with multiple owners, including larger ones.

[5] Persons engaged include both workers and unpaid working owners.

[6] Combining information from World Bank Enterprise Surveys with follow-up interviews of entrepreneurs in five African countries, Aterido and Hallward-Driemeier (2011) find no significant gender gaps in productivity when the standard 'female participation in ownership' (based on capital shares) is used. However, when the actual primary decision-maker is female, firms do exhibit significantly lower productivity.

Some studies, on the other hand, have had the advantage of using purposely collected survey data including specific questions related to the effective control of women in the decision-making process (see Aterido and Hallward-Driemeier 2011 on SSA firms; and Presbitero et al. 2014 on Latin American firms). Unfortunately, our own dataset on the Ethiopian manufacturing sector does not contain information on firms' ultimate decision-makers, to the extent that those are different from the owners. Rather, our data allows us to construct a measure of female ownership based on the degree of control exercised by women both in terms of a firm's current capital shares and the composition of the corporate board. Importantly, Sekkat et al. (2015) demonstrate that developing country firms with women among their shareholders are more likely to have a female CEO too. For African firms, it is found that the higher the female ownership share, the stronger its effect on the likelihood of having a female CEO.

Our preferred measure of female ownership is based on a relatively restrictive definition, i.e. whether women hold at least 50% of the current capital of firms. It is on the basis of such definition that we present some basic descriptive statistics and run our baseline regressions. Alternative definitions of female ownership will be used to check the robustness of our first results further on in the paper.

Whereas the number of firms increased substantially over the period considered (more than doubling from 2003 to 2009), the share of female-owned firms remained fairly stable, around 12% (cf. Table 1).[7]

TABLE 1— SAMPLE DISTRIBUTION OF FIRMS, BY OWNERSHIP

| Year | N. of firms | % of female owned |
|---|---|---|
| 2003 | 795 | 12.70% |
| 2004 | 857 | 13.42% |
| 2005 | 715 | 9.79% |
| 2006 | 1,007 | 11.92% |
| 2007 | 1,206 | 11.86% |
| 2008 | 1,577 | 12.43% |
| 2009 | 1,792 | 12.22% |

***Source***: Authors' elaborations on CSA manufacturing census data.
*Note*: In 2005 (Ethiopian year 1997), the Census was run as a representative survey.

Compared to their male-owned counterparts, female-owned firms are generally more concentrated, both geographically (almost 60% in the administrative region of Addis Ababa – which in turn accounts for about 49% of all firm-years in the sample) and, especially, at the sectoral level (more than 60% in the food processing and mineral products industries, dominated by bakeries and firms producing concrete, respectively) (cf. Tables A1 and A2 in the Appendix). In addition, and in correspondence with existing evidence from other developing countries, including in SSA, female-owned firms are smaller in size, less capital-intensive and less internationalised (i.e., engaged in

[7] This share raises to 29% of the total sample if we consider a less restrictive definition (i.e., at least one woman in the board) and reduces to 7.2% in case female ownership is defined as 100% of capital being held by women.

exporting and/or importing) compared to male-owned firms. Moreover, on average, they employ larger shares of female workers overall and of *skilled* female workers (cf. Table 2).

TABLE 2— BASIC CHARACTERISTICS OF FIRMS, BY OWNERSHIP

| Variable | Male-owned | Female-owned |
|---|---|---|
| Number of employees | 86.116 | 35.679 |
| Sales (log) | 13.849 | 13.045 |
| Value added (log) | 13.776 | 12.958 |
| Fixed assets to number of employees (log) | 9.401 | 8.894 |
| Female/male workers (ratio) | 0.566 | 0.816 |
| Female/male skilled workers (ratio) | 0.545 | 0.824 |
| Exporter (dummy) | 0.0504 | 0.018 |
| Importer (dummy) | 0.676 | 0.595 |
| Foreign-owned (dummy) | 0.044 | 0.003 |

***Source*:** Authors' elaborations on CSA manufacturing census data.

## *Productivity estimates*

The main indicator we use to measure the relative performance of male- and female-owned firms in this paper (as in the literature) is productivity. We focus on TFP, an indicator widely employed in the literature on heterogeneous firms (Melitz 2003; Helpman et al. 2004). The production function is assumed to take the form of a standard Cobb-Douglas specification:

$$Y_{it} = A_{it} L_{it}^{\alpha_L} K_{it}^{\alpha_K} M_{it}^{\alpha_M}, \qquad \alpha_L, \alpha_K, \alpha_M > 0$$

where $Y_{it}$ stands for the output of firm $i$ in year $t$; $L_{it}$, $K_{it}$, $M_{it}$ are the inputs in the form of labour, capital and intermediate inputs; $A_{it}$ is the Hicks-neutral efficiency level, which represents the TFP of firms; and the $\alpha$'s are factor shares. At the firm level, $A$ includes not easily measurable factors, such as R&D stocks, technology, quality and marginal efficiency (Del Gatto et al. 2011).

A transformation into logarithms allows one to introduce a linear estimation of the production function (with small letters representing logs):

$$y_{it} = \beta_0 + \beta_l l_{it} + \beta_k k_{it} + \beta_m m_{it} + v_{it} + \pi_{it}$$

where the error term has two components: $v_{it}$, which represents the level of productivity of the firm, and $\pi_{it}$, the i.i.d. component that is uncorrelated with input choices. $v_{it}$ is the key variable to be computed after having estimated the production function and solved for $\omega_{it}$ as the standard Solow residual:

$$\widehat{\omega}_{it} = \hat{v}_{it} + \hat{\beta}_0 = y_{it} - \hat{\beta}_l l_{it} - \hat{\beta}_k k_{it} - \hat{\beta}_m m_{it}$$

Considering that $\widehat{\omega}$ is observed by firms and influences their choice of inputs, thus making the error term correlated with the independent variables and rendering the coefficients of a standard OLS model biased, alternative methods to estimate TFP have been proposed in the literature, including fixed effects and system-GMM estimators (see Del Gatto et al. 2011; Van Beveren 2012 for reviews). More consistent approaches include those adopting semi-parametric estimators using proxies to correct for unobservable productivity shocks and the potential simultaneity bias in the choice of input levels. In what follows, we focus on the approach proposed by Olley and Pakes (1996), which controls not only for the simultaneity bias, but also for the potential selection bias resulting from the relationship between productivity shocks and the probability of firms exiting the market (which would bias the coefficient of capital downward if not properly accounted for). More specifically, the method of Olley and Pakes (1996) solves the simultaneity bias by using investment to proxy for unobserved time-varying productivity shocks, while the selection problems are addressed based on survival probabilities.

We estimate TFP separately for each industry, identified by its 2-digit ISIC code. Output is measured by value added (calculated as total sales minus costs), labour by the total number of employees, capital is the book value of fixed assets, and intermediate inputs are proxied by the sum of all costs related to the inputs used in the production process. Variables reported in monetary terms have been deflated using Ethiopia's GDP deflator, obtained from the IMF's World Economic Outlook database, with 2005 as the base year.

## Regression results

First, we are interested in evaluating whether the productivity gap that has been found in most existing studies on female-owned firms is also present in our census data on Ethiopian manufacturing firms. We estimate regressions of the following form:

$$\widehat{\omega}_{it} = \mu + \gamma female_{it} + \sum \eta X_{it} + \delta_j + \theta_x + \lambda_t + \varepsilon_{it}$$

where $\omega_{it}$ is the log TFP; *female* is our variable of interest, i.e., a dummy identifying female ownership; $X_{it}$ is a vector of control variables (see further); $\delta_j$, $\theta_x$, and $\lambda_t$ are industry, region and year fixed effects, respectively; and $\varepsilon_{it}$ is the error term. Standard errors are clustered at the firm level. Importantly, our identification strategy is based on within-industry variation in productivity according to the ownership of firms. Within-firm changes in ownership are relatively rare over the period considered and do not provide sufficient variation to interpret the effect of a switch from male to female ownership on productivity.[8]

[8] Only about 4% of firms switched from male to female ownership over time, according to our preferred definition.

### *Unconditional regression estimates*

Table 3 summarises the results of a first set of unconditional regressions linking female ownership to TFP.

TABLE 3—RESULTS, OLS ESTIMATOR

| VARIABLES | (1)<br>50% ownership | (2)<br>50% ownership | (3)<br>Any female participation in ownership | (4)<br>100% ownership | (5)<br>voting rights |
|---|---|---|---|---|---|
| female | -0.213** | -0.292*** | | | |
| | (0.104) | (0.0676) | | | |
| female_owned | | | -0.140*** | | |
| | | | (0.0464) | | |
| female_ownership100 | | | | -0.513*** | |
| | | | | (0.0926) | |
| female_own_seat | | | | | -0.280*** |
| | | | | | (0.0681) |
| Constant | 10.19*** | 10.21*** | 10.24*** | 10.21*** | 9.994*** |
| | (0.0502) | (0.0767) | (0.0776) | (0.0769) | (0.0801) |
| Observations | 7,727 | 7,726 | 7,726 | 7,726 | 6,811 |
| R-squared | 0.001 | 0.655 | 0.654 | 0.657 | 0.632 |
| Industry FE | NO | YES | YES | YES | YES |
| Region FE | NO | YES | YES | YES | YES |
| Year FE | NO | YES | YES | YES | YES |

*Notes*: *** p<0.01, ** p<0.05, * p<0.1. Standard errors are clustered at the firm level. The dependent variable is log TFP. Our independent variable of interest in columns (1)-(2) is a dummy taking a value 1 if female-owned capital is larger than 50% of total; in column (3) is a dummy taking a value 1 if there is at least 1 woman among the owners; in column (4) is a dummy taking a value 1 if female-owned capital is 100% and in column (5) is a dummy taking a value 1 if there is a majority (50%) of women among the owners of the firm.

The results are in line with extant evidence from other developing countries and show that, depending on the specification adopted, the productivity difference between female and male firms in Ethiopia ranges from 13% to 40% (calculated as $e^{-0.140} - 1$ and $e^{-0.513} - 1$), with a gap of 25% ($e^{-0.292} - 1$) in a specification including a full set of industry, region and year fixed effects.

The size of the productivity gap is relatively sensitive to the exact definition of female ownership. In fact, the size of the estimated gap using our preferred definition of female ownership based on a majority (50% or more) share of firm capital lies in between the gaps for a broader definition based on the presence of at least one woman among the owners (column 3), and for the most restrictive definition, i.e., fully female-owned firms (column 4).[9] This is consistent with the large variability of estimates found by other studies, and it seems to confirm that what really matters to identify differences in firm performance is the extent of decision-making power, rather than ownership per se (see also Aterido and Hallward-Driemeier 2011). While this distinction is less meaningful for studies looking at microenterprises where the two concepts tend to overlap (de Mel

[9] Importantly, the results remain basically unchanged when using an alternative definition of majority in ownership based on having more than 50% of women among the total number of owners (Column 5).

et al. 2009; Bernhardt et al. 2017), it becomes more important when considering more structured firms like the ones we analyse in this paper.

### *Conditional regression estimates and potential mechanisms*

Having investigated the unconditional relation between female ownership and productivity, we now take the analysis further and try to improve our understanding of the origins of the observed gender productivity gap. First, we add a set of control variables likely to affect productivity levels that are typically used in heterogeneous firm models (Helpman et al. 2004). These include firm size, defined as the log number of total employees (*empl*); the age of the firm, i.e. the log number of years since its first establishment (*age*); capital intensity, measured as the log of the ratio of fixed assets to the number of employees (*kl*); and two dummies controlling for the status of firms as *exporter* and/or *importer*. Second, we attempt to explore the main mechanisms leading to lower productivity of female-owned firms by interacting our female ownership dummy with a range of firm- and industry-specific factors.

The results of these additional regressions are reported in Table 4. All standard controls behave as expected, in accordance with the provisions of heterogeneous firms models (Helpman et al. 2004). Previous evidence on the productivity gap is confirmed. As expected, the inclusion of firm-specific controls reduces the size of the gap, the estimated difference in productivity now being close to 12% ($e^{-0.125}$ - 1), but its statistical significance remains high.

In the remaining columns (2-7) of Table 4, we explore some of the potential mechanisms at stake by adding selected interaction terms. To ease interpretation, we have demeaned variables before interacting them with the female ownership dummy; the coefficient of the dummy can thus be read as the effect of female ownership when the interaction variable is evaluated at its average value.

Starting from column 2, looking at capital intensity, we find that the observed gender gap in productivity increases with the use of capital. This is in line with existing literature emphasizing lower returns to capital for female-owned firms (though all these studies look at household-based microenterprises; see e.g. de Mel et al. 2008, 2009), and therefore a dimension deserving more attention. Further results, reported in the remainder of this section and in the section on decomposition analysis confirm and help to better qualify this result.

In columns 3 and 4 we do not find any evidence of specific mechanisms linking the productivity gap to either the size of the firm or to their access to finance. Our proxy for the latter is a dummy (*ir*) taking the value of 1 if the firm reports positive values of interests paid (as in Shiferaw 2016, who uses the same census data). While we show that firms accessing finance tend to display higher productivity, the interaction term (*fem*ir*, column 4) is not statistically significant, implying that female-owned firms using credit are, ceteris paribus, not necessarily less productive than male firms doing so.

TABLE 4—RESULTS, OLS WITH CONTROLS AND INTERACTION TERMS

| VARIABLES | (1) baseline | (2) kl | (3) size | (4) finance | (5) female_workers | (6) lab_intensity | (7) competition |
|---|---|---|---|---|---|---|---|
| female | -0.125** | -0.154** | -0.115* | -0.122 | -0.0911 | -0.124** | -0.120* |
| | (0.0615) | (0.0647) | (0.0586) | (0.0762) | (0.0629) | (0.0544) | (0.0614) |
| age | 0.103** | 0.101** | 0.104** | 0.103** | 0.0745* | 0.110*** | 0.111*** |
| | (0.0413) | (0.0412) | (0.0412) | (0.0411) | (0.0415) | (0.0383) | (0.0414) |
| empl | 0.392*** | 0.390*** | 0.389*** | 0.370*** | 0.404*** | 0.384*** | 0.395*** |
| | (0.0224) | (0.0222) | (0.0220) | (0.0228) | (0.0226) | (0.0202) | (0.0225) |
| kl | 0.0547*** | 0.0689*** | 0.0550*** | 0.0473*** | 0.0504*** | 0.0520*** | 0.0558*** |
| | (0.0120) | (0.0132) | (0.0119) | (0.0120) | (0.0118) | (0.0113) | (0.0120) |
| exporter | 0.342*** | 0.342*** | 0.344*** | 0.353*** | 0.329*** | 0.341*** | 0.360*** |
| | (0.111) | (0.110) | (0.111) | (0.111) | (0.110) | (0.106) | (0.111) |
| importer | 0.271*** | 0.268*** | 0.270*** | 0.265*** | 0.267*** | 0.245*** | 0.265*** |
| | (0.0424) | (0.0423) | (0.0423) | (0.0421) | (0.0422) | (0.0402) | (0.0424) |
| fem_kl | | -0.0563** | | | | | |
| | | (0.0228) | | | | | |
| fem_empl | | | 0.0281 | | | | |
| | | | (0.0703) | | | | |
| fem_ir | | | | -0.000862 | | | |
| | | | | (0.113) | | | |
| ir | | | | 0.185*** | | | |
| | | | | (0.0399) | | | |
| fem_share_n_female | | | | | -0.109** | | |
| | | | | | (0.0512) | | |
| n_female | | | | | -0.0683** | | |
| | | | | | (0.0328) | | |
| fem_lab_intensity | | | | | | 0.143** | |
| | | | | | | (0.0682) | |
| lab_intensity | | | | | | -0.238*** | |
| | | | | | | (0.0654) | |
| fem_hh | | | | | | | 0.157 |
| | | | | | | | (0.206) |
| hh | | | | | | | -0.407*** |
| | | | | | | | (0.102) |
| Constant | 7.887*** | 8.407*** | 9.175*** | 7.961*** | 7.947*** | 7.971*** | 7.904*** |
| | (0.175) | (0.130) | (0.181) | (0.176) | (0.174) | (0.169) | (0.176) |
| Observations | 7,546 | 7,546 | 7,546 | 7,546 | 7,530 | 7,544 | 7,546 |
| R-squared | 0.719 | 0.720 | 0.719 | 0.721 | 0.722 | 0.751 | 0.720 |

*Notes*: *** p<0.01, ** p<0.05, * p<0.1. Standard errors are clustered at the firm level. The dependent variable is log TFP. All regressions include industry, region and year fixed effects. Apart from the interest payments dummy (*ir*), variables interacted with female ownership have been first demeaned.

An important result is the one reported in column (5) and has to do with the role of female workers employed by the firm (*n_female_male,* a variable measuring the ratio of female to male workers within each firm). The interaction term (*fem*n_female_male*) appears to indicate that the productivity gap can partly be attributed to those female-owned firms employing a large share of female workers, a factor that in itself is also found to be negatively correlated with productivity. This result looks consistent with evidence reported by Hellerstein and Neumark (1999) on Israeli's manufacturing firms, but opposite to what Flabbi et al. (2016) find in a panel of Italian manufacturing firms. In their theoretical setting, Flabbi et al. (2016) claim that female executives are better at processing information on female workers, resulting in reduced discrimination (e.g., in wages) and, ultimately, improved firm performance (due to better matching of skilled female workers) when the share of female workers is higher. This does not seem to be the case in Ethiopia. Rather, our result likely derives from the clustering of female workers and owners in low-productivity activities (Juhn et al. 2014; Kucera and Tejani 2014).

In column (6), an industry-level measure of labour intensity (*labour_intensity*, constructed from our census data by calculating each 4-digit industry's share of total wages over capital) is interacted with female ownership (*fem*lab_intensity*). The positive coefficient of the interaction term is consistent with what we just discussed regarding firms' capital intensity (column 2), as it shows that the productivity disadvantage of female-owned firms tends to be attenuated in more labour-intensive industries.

A last finding from Table 4 relates to the role of competition. When we add a variable measuring competition at the sectoral and regional level by means of an Herfindal index (*hh*) for firms' sales, we do not see significantly different gender productivity gaps in more competitive markets (column 7); unlike what is suggested by some studies reporting greater aversion of female owners towards competition and risk (Niederle and Vesterlund 2007). Since this is another dimension deserving more attention, we have run a set of additional (probit) regressions looking at gender related differences in the probability of firm exit. Results, reported in Table A3 in the Appendix, show that the probability of exit of female-owned firms is not statistically different from the one of their male counterparts, even after conditioning on observable firm characteristics and accounting for a possible bias due to the use of a survey – rather than a census – in the year 2005. Further analysis complementing these findings is included in the section on differences in productivity distributions.

## Results on firm productivity distributions

In this section, we move beyond *average* gender gaps in firm productivity and analyse differences between the respective productivity *distributions* of male- and female-owned firms. We apply a quantile decomposition approach developed by Firpo et al. (2007) to distinguish between compositional and structural effects in explaining gender productivity gaps at different points of the overall distribution. Next, we apply the methodology developed by Combes et al. (2012) to conduct a formal comparison of productivity distributions between female- and male-owned firms in our Ethiopian census data.

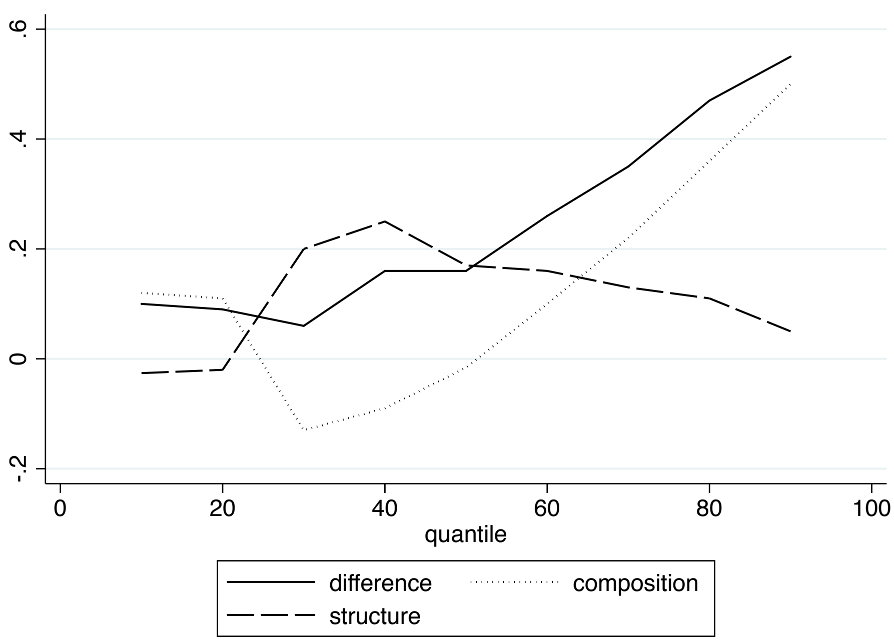
### *Decomposition analysis*

The decomposition method developed by Firpo et al. (2007) allows us to look at the productivity gap between male- and female-owned firms at different points in the productivity distribution and to identify how much of the gap can be explained by differences in observable characteristics or, alternatively, by differences in returns to the same set of characteristics. Using their terminology, the former is defined as the *composition* effect and the latter as the *structural* effect.

Importantly, this methodology generalises the standard Oaxaca-Blinder decomposition to different moments of the distribution. This is done by first reweighting the distribution (of male-owned firms in our case) to account for the composition and structural effects separately. Second, recentered influence functions (RFI) are used to estimate the dependent variable at each quantile of the distributions. These estimates are then used to run the decomposition analysis. A more detailed description of the methodology can be found in the Firpo et al. (2007; 2009), whereas an application of the methodology on a research question similar to ours can be found in the study by Nix et al. (2016), which shows that a significant part of the performance gap of female-owned microenterprises in four SSA countries can be explained by structural effects.

Results of the decomposition analysis are summarised in Figure 1. Figure 1 shows differences in firm productivity across gender, and decomposes them into composition and structural effects at each decile of the productivity distribution. It shows that differences in productivity increase moving towards the upper tails of the distribution (where female-owned firms are less represented), and that in such tails composition effects have the highest explanatory power. Conversely, structural effects are largely responsible for the productivity differences found in the central part of the distribution, i.e. from the 3rd to the 6th decile. No systematic differences among the two types of effects are observed in the bottom deciles.

FIGURE 1. RESULTS DECOMPOSITION ANALYSIS




*Notes*: This Figure shows male-female differences in firm productivity (in log TFP) at each decile of the productivity distribution, and their decomposition into (observed) compositional and (unobserved) structural effects according to the approach of Firpo et al. (2007, 2009).

The approach also allows to disentangle the contribution of the individual variables (Table A4). Importantly, among the central quantiles of the distribution the role of capital appears to be crucial. Returns to capital, in particular, account for most of the unobserved part of the productivity differences, since it is often the only significant covariate when estimating the structural effect across its components. On the other hand, observable characteristics related to the size, capital and internationalization (especially the importer status) of firms are the main factors accounting for the larger male-female productivity differences linked to the composition effect that we observe when moving towards the upper tails of the distribution.

### *Differences in productivity distributions*

Combes et al. (2012) study the positive correlation between city size and firm productivity and present a quantile-based approach to discriminate empirically between two common explanations, i.e., firm selection and agglomeration economies. Supported by a theoretical model incorporating both firm entry/exit and between-firm interactions, they show that stronger firm selection in larger cities should result in a left truncation of the productivity distribution of firms active there; whereas stronger agglomeration effects should lead to a right-shifted and, if more productive firms are also better able to exploit such agglomeration economies, more dilated productivity distribution. Since making proper distinctions between shift, dilation and truncation by visually comparing two distributions may be difficult, Combes et al. (2012) translate their theoretical framework into a quantile-based estimator that allows them to formally identify shift, dilation and truncation parameters.[10] It is this estimator that we transplant to our specific context. As pointed out by Kondo (2017), the Combes et al. (2012) quantile approach can be applied more generally, outside the fields of urban economics and economic geography, to compare any two (productivity) distributions.[11]

The main value added of the Combes et al. (2012) approach over the traditional regression results we presented previously is that it provides additional insights on some of the potential mechanisms underlying the average productivity gap of female-owned firms. Are productivity gaps systematically observed over the whole distribution, and/or are they reinforced by differences between the top-performing female- and male-owned firms? And to what extent are productivity gaps also explained by gender differences in the survival of the least productive firms? Unlike the decomposition methodology of Firpo et al. (2007), which considers the influence of covariates at different quantiles of the distribution, the Combes et al. (2012) quantile-based method summarises

[10] In broad terms, the estimation approach minimises the errors in matching the quantiles of the two distributions concerned and relies on a two-step-iterated numerical optimization (with iterations over the truncation parameter and over combinations of the shift and dilation parameters). For more technical details, see Combes et al. (2012) and Kondo (2017).

[11] That notwithstanding, most papers employing the Combes et al. (2012) methodology stay close to the original set-up of comparing firm productivity between larger and small cities (see, e.g., Acetturo et al. 2014; Kondo 2016). We are not aware of other studies using it to compare firm productivity along gender dimensions.

the differences in productivity distributions of male- and female-owned firms in three transformation parameters (shift, dilation and truncation), thus providing a complementary perspective. We further lay out the logic of the Combes et al. (2012) approach and how it applies to our context in Appendix B.

Given our regression results on the productivity gap in presented previously, we expect the TFP distribution of female-owned firms to be *left*-shifted relative to the male distribution ($A<0$). Besides such a general productivity disadvantage, we may also expect relatively stronger homogeneity in productivity among female firms, due to their concentration in low-productivity industries, their smaller scale and lower capital intensity. Ceteris paribus, this should be reflected in a less dilated TFP distribution for female- than for male-owned firms ($D<1$). Finally, we do not have very strong priors on the relative truncation parameter, especially since we do not find significant gender differences in firm exit (cf. Appendix Table A3).

Table 5 presents the parameters for relative shift ($A$), dilation ($D$) and truncation ($S$) for the female- vs. male-owned firm productivity distributions estimated using Combes et al.'s (2012) quantile approach.[12] Productivity is here defined as log TFP averaged for each firm over the available (maximum 7) years of data. The reported standard errors are obtained using 500 bootstrap replications and significance is based on two-sided z-tests of the respective null hypotheses that $A$=0, $D$=1, and $S$=0.[13]

As can be seen in the first row of Table 5, when we consider our full sample of female- and male-owned manufacturing firms we find a negative value for $A$ of about -0.140, which is also highly significant. This means that in order to move from the male to the female average TFP distribution one needs to shift the former distribution leftwards (cf. Figure 2) and, on its own (without considering dilation and truncation), implies a decrease in mean productivity of just over 13% ($e^{-0.140} - 1$). Moreover, in the full sample, $D<1$, meaning that the female firms' TFP distribution is

[12] As in Combes et al. (2012), we trim the extreme (bottom/top 1%) values of firm-level TFP, separately for female- and male-owned firms, to obtain more reliable and unbiased estimates of the parameters $A$, $D$ and $S$.

[13] To produce these results, we have used the *estquant* command in Stata developed by Kondo (2017). As a robustness check, we have also performed the same analysis using the original SAS code of Combes et al. (2012). Both codes yield very similar point estimates for $A$, $D$ and $S$. However, the latter requires much longer computational time for large number of bootstrap iterations. To deal with the small subsample of female firms (about 250), bootstrap sampling is stratified on the gender of ownership and bootstrap samples where less than 50% of the original female firms are represented are discarded. This ensures that our bootstrap samples include a sufficient number of unique female firms. Further stratification of bootstrap sampling on sectors or regions is employed in view of large sectoral/regional heterogeneity in productivity. See notes below Table 5 for details. As in Combes et al. (2012), we also report in Table 5 a measure of explanatory power, which is calculated as

$$R^2 = \frac{M(\hat{A}, \hat{D}, \hat{S})}{M(0,1,0)}$$

where $M$ is the criteria function minimised by Combes et al.'s (2012) estimator. In other words, this pseudo $R^2$ indicates how much of the mean-squared quantile difference between the female and male firm productivity distributions is explained by the three parameters.

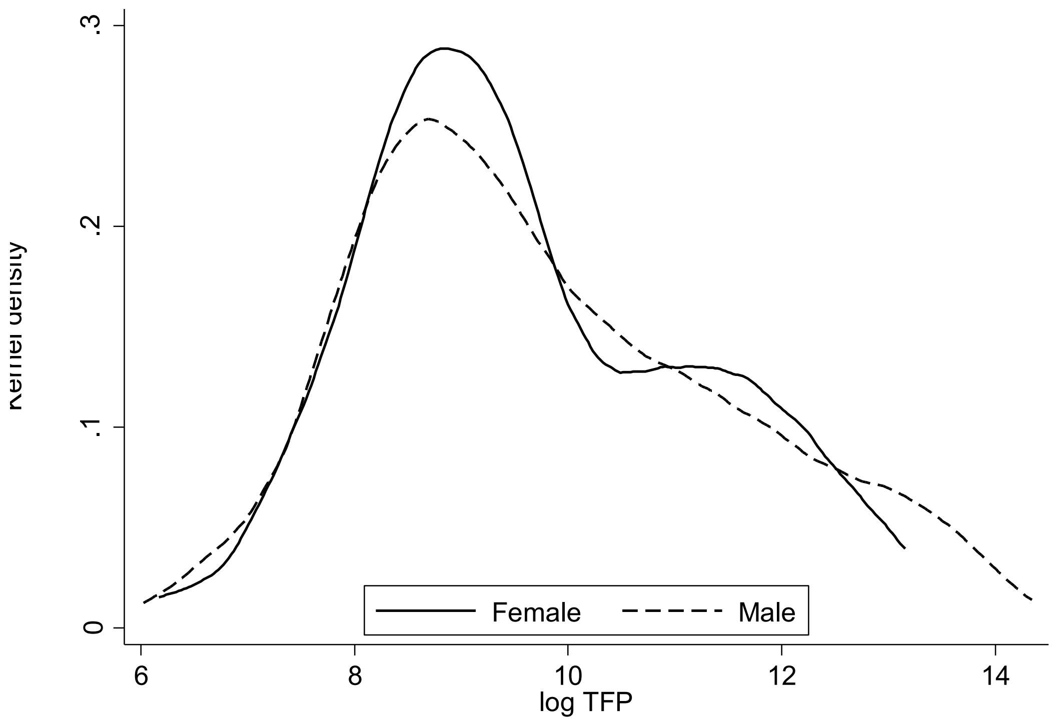
less dilated than the male distribution. This is a relevant property, since it reflects the relative homogeneity of female-owned firms, which appear to be less likely to succeed in achieving high productivity than male firms. Finally, we also find evidence of some (statistically significant but economically small) truncation differences in the full sample. More specifically, we uncover an S<0, suggesting that, overall, female firms are more likely to stay in the market even at lower productivity levels, compared to male firms. This latter finding seems to be in line with the results of a survival analysis based on the same Ethiopian census data that we employ here (though based on an earlier period, see Shiferaw 2009). While it is not possible to identify the exact underlying mechanisms with the data at hand, one possible interpretation could be a relatively more cautious approach to business by female owners (Niederle and Vesterlund 2007). Another interpretation is that the high gender discrimination in the Ethiopian labour market perhaps provides an extra incentive for women to keep running their own businesses, even when they are comparatively unproductive.

TABLE 5—RESULTS, COMPARISON OF PRODUCTIVITY DISTRIBUTIONS

| # | Category/sector | Shift (A) | Dilation (D) | Truncation (S) | R2 | Obs. |
|---|---|---|---|---|---|---|
| 1 | Female-owned vs male-owned, full sample | -0.140*** | 0.855*** | -0.0115*** | 0.867 | 2465 |
| | | (0.0089) | (0.0037) | (0.0012) | | |
| 2 | Female-owned vs male-owned, Addis Ababa only | -0.306*** | 0.872*** | 0.00213 | 0.902 | 998 |
| | | (0.0126) | (0.0049) | (0.0012) | | |
| 3 | Female-owned vs male-owned, food and beverages only (ISIC 15) | -1.232*** | 0.973 | -0.00425 | 0.992 | 561 |
| | | (0.1525) | (0.0676) | (0.0886) | | |
| 4 | Female-owned vs male-owned, mineral products only (ISIC 26) | -0.105*** | 0.857*** | -0.0174* | 0.500 | 682 |
| | | (0.0165) | (0.0207) | (0.0083) | | |
| 5 | Female-owned vs male-owned, full sample (Residual distribution, controlling for sectors) | -0.386*** | 1.098*** | 0.00442 | 0.894 | 2465 |
| | | (0.0152) | (0.0205) | (0.0039) | | |
| 6 | Female-owned vs male-owned, full sample (Residual distribution, controlling for sectors, number of employees and capital intensity) | -0.226*** | 1.102*** | -0.00345*** | 0.949 | 2446 |
| | | (0.0065) | (0.0043) | (0.0010) | | |

*Notes*: Results are based on a quantile approach developed by Combes et al. (2012) (see the section outlining results on firm productivity distributions) and use average log TFP as productivity measure. Bootstrapped standard errors (using 500 replications) in parentheses. Bootstrap sampling is stratified on gender of ownership and on either sector (lines 1 and 2) or region (lines 3, 4, 5 and 6) and uses only samples where at least 50% of the original female-owned firms are represented.[14] *** p<0.01, ** p<0.05, * p<0.1.

FIGURE 2. FEMALE- VS. MALE-OWNED FIRM PRODUCTIVITY DISTRIBUTIONS, FULL SAMPLE




*Notes*: Densities of log TFP distributions are estimated using an Epanechnikov kernel function. Top and bottom 1% of log TFP distributions are trimmed (separately for male and female distributions).

Overall, these results very well complement and further qualify those presented in the section on decomposition analysis, especially for what concerns the tails of the distribution. In Table A5 in the Appendix we report some descriptive statistics on the firms populating the top and bottom 10% of the overall productivity distribution differentiating by gender of the owners. Male- and female-owned firms at the bottom of the productivity distribution look relatively similar as far as main endowments are concerned. This is consistent with the small difference in compositional effects at the first decile observed in Figure 1. Two notable exceptions are that female owned firms are largely employing female workers and that they are operating at a very low scale of activity. The latter could perhaps explain the larger persistence of female firms running their business despite low scale/profitability, as indicated by the longer left tails of their productivity distribution relative to male firms. At the top 10% of the distribution, on the contrary, marked gender differences are observed in almost all observable firm characteristics, including in the number of employees, capital endowments and importer status, again consistent with the findings of the decomposition analysis (cf. Figure 1). The few female-owned firms that do make it to the top 10% of most productive firms do so in spite of large differences in the observables. The very small number of such 'outstanding' female-owned firms likely explains part of the lower distribution dilation (relative to male-owned firms) that we documented using the Combes et al. (2012) approach.

*Further results based on the productivity distributions*

On the second row of Table 5 we replicate our Combes et al. (2012)-based analysis focusing on those firms based in the neighbourhood of the capital, Addis Ababa (cf. Figure A1 in Appendix). Our choice to consider Addis Ababa separately is motivated by the very strong presence of manufacturing firms in the Addis region (about half of our sample) and by the literature emphasizing the specific productivity advantages of being located in larger agglomerations (Combes et al. 2012; Kondo 2016), including in SSA (Sanfilippo and Seric 2016).[15] The earlier-found left shift in the female productivity distribution is economically more significant in Addis Ababa than in the full sample (implying a 26% lower TFP at the mean) and also the lower dilation is again clear. On the other hand, however, the truncation parameter is no longer statistically significant. We thus conclude that there are no strong survival differences between Addis Ababa-based female- and male-owned firms.

In rows 3 and 4 of Table 5, we estimate the three transformation parameters for the TFP distributions of firms within the food processing and mineral products industries, respectively (cf. Figures A2 and A3 in Appendix). These are two manufacturing industries where female owned-firms are relatively well represented (cf. Table A2).[16] As in the full sample, we again observe, in both sectors, relative left shifts, lower dilation and less left-truncation for the female versus male firm productivity distributions. Yet, there is some heterogeneity, as the shift is particularly strong for the labour-intensive food processing industry while dilation (and, to a lesser extent, truncation) are observed in the mineral products indsutries.

Finally, we perform two additional exercises on the full sample: we regress log TFP on either industry dummies or industry dummies, firm size (number of employees) and capital intensity, and then apply the Combes et al. (2012) approach to the regression residuals. The results are shown in rows 5 and 6 of Table 5 (cf. also Figures A4 and A5 in Appendix). We again find a significant relative left shift of the female firms' productivity distribution, but one that is economically larger than in the unconditional estimates of row 1. The earlier truncation differences disappear. And, interestingly, once sectoral composition is controlled for, the productivity distribution of female firms is actually slightly *more* dilated than that of their male counterparts (i.e., D>1). This suggests that the relative homogeneity (smaller dispersion) in female firms' productivity overall relates to their concentration in certain lower-productivity sectors (most notably, the food processing and mineral products industries).

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[15] For the specific case of Ethiopia, Rijkers et al. (2010) carry out a rural–urban comparison of manufacturing firms and find that the benefits associated with agglomeration are 'concavely' related to city sizes. They argue that firms in remote rural areas are much less productive than firms located elsewhere but fail to uncover big productivity differences between rural towns and major urban areas. On the other hand, Siba et al. (2012) analyze the effect of firm clustering on output prices and productivity in Ethiopia and find a positive and statistically significant relationship between the density of firms that produce a given product and productivity.

[16] As noted before, female-owned firms in these two industries are mostly bakeries and firms producing concrete.

## Concluding remarks

In this paper, we have investigated whether and how productivity differs between female- and male-owned firms in Ethiopia. Using census data on manufacturing firms over the 2003-2009 period, we provide robust evidence of a productivity disadvantage for female-owned firms, which can be plausibly linked to factors such as their size, endowments (of capital in particular), differences in returns to those endowments (again, primarily returns to capital), and specialization/sorting into specific less productive sectors.

Our findings underline the importance of moving beyond standard analysis based on the average productivity gap. The various methods we apply demonstrate how there might be different gaps, diverse mechanisms leading to them, as well as different explanations for their existence. For instance, based on formal comparisons of productivity distributions, we show that one important mechanism underlying the gender-based firm productivity gap is a higher homogeneity in terms of productivity, coupled with the relative lack of highly productive female-owned firms.

One issue that also emerges from our distributional analyses is that whereas observed firm-level characteristics can account for much of the gaps recorded at the extreme tails of the productivity distribution, non-negligible productivity differences found in the more central part of the distribution, tend to derive from other factors, including in particular returns to capital, that are more difficult to catch with our data. Further research, including more in-depth, qualitative work, is therefore needed to get a fuller understanding of the key factors explaining the gap in performance of female-owned firms in Ethiopia.

Still, we believe our findings so far provide valuable policy lessons for a country like Ethiopia, currently embarked on a process of structural transformation. Productivity growth is crucial in such process but cannot realise its full potential unless the whole productivity distribution, including that of female-owned firms, shifts rightwards. Indeed, the literature on resource misallocation shows that persistent productivity gaps among firms result in lower aggregate productivity and weigh on overall economic growth (Hsieh and Klenow 2009). Targeting the productivity growth of female-owned firms therefore also seems relevant for purposes of allocative efficiency.

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## Appendix A: Tables and figures

TABLE A1—DISTRIBUTION OF FIRMS BY REGION AND OWNERSHIP (% OF TOTAL)

| | male-owned | female-owned | total |
|---|---|---|---|
| tigray | 10.25 | 4.47 | 9.55 |
| afar | 0.52 | 0.42 | 0.5 |
| amhara | 9.36 | 9.24 | 9.35 |
| oromia | 16.94 | 14.75 | 16.67 |
| somalie | 0.59 | 0.31 | 0.55 |
| benshangul | 0.23 | 0.1 | 0.21 |
| s.n.n.p.r. | 11.11 | 5.71 | 10.46 |
| gambela | 0.24 | 0.52 | 0.28 |
| harari | 1.26 | 2.18 | 1.37 |
| addis ababa | 47.69 | 59.92 | 49.17 |
| dire dawa | 1.82 | 2.39 | 1.89 |

***Source:*** Authors' elaborations on CSA manufacturing census data. SNNPR stands for Southern Nations, Nationalities and Peoples. Ownership based on whether or not female-owned capital is larger than 50% of total.

TABLE A2—DISTRIBUTION OF FIRMS BY INDUSTRY (% OF TOTAL)

| ISIC rev. 3 | Industry | Male-owned | Female-owned | Total |
|---|---|---|---|---|
| 15 | Manufacturing of food products and beverages | 24.41 | 34.85 | 25.68 |
| 26 | Manufacturing of other non-metallic mineral products | 18.91 | 26.24 | 19.80 |
| 22 | Publishing, printing and reproduction of recorded media | 6.18 | 8.51 | 6.47 |
| 36 | Manufacturing of furniture; manufacturing n.e.c. | 15.93 | 8.09 | 14.98 |
| 20 | Manufacturing of wood and of products of wood and cork, except furniture | 2.06 | 3.53 | 2.24 |
| 25 | Manufacturing of rubber and plastics products | 5.11 | 3.53 | 4.92 |
| 19 | Tanning and dressing of leather; manufacturing of luggage, handbags, saddlery, harness and footwear | 5.97 | 3.11 | 5.62 |
| 28 | Manufacturing of fabricated metal products, except machinery and equipment | 6.40 | 3.11 | 6.00 |
| 24 | Manufacturing of chemicals and chemical products | 5.21 | 2.70 | 4.91 |
| 17 | Manufacturing of textiles | 2.93 | 2.28 | 2.86 |
| 18 | Manufacturing of wearing apparel; dressing and dyeing of fur | 2.69 | 1.45 | 2.54 |
| 29 | Manufacturing of machinery and equipment n.e.c. | 0.46 | 1.14 | 0.54 |
| 21 | Manufacturing of paper and paper products | 1.06 | 1.04 | 1.06 |
| 27 | Manufacturing of basic metals | 1.17 | 0.21 | 1.06 |
| 34 | Manufacturing of motor vehicles, trailers and semi-trailers | 1.26 | 0.21 | 1.13 |
| 16 | Manufacturing of tobacco products | 0.10 | 0.00 | 0.09 |
| 30 | Manufacturing of office, accounting and computing machinery | 0.01 | 0 | 0.01 |
| 31 | Manufacturing of electrical machinery and apparatus n.e.c. | 0.11 | 0 | 0.11 |

***Source*:** Authors' elaborations on CSA manufacturing census data.
Notes: Industries are reported at the 2-digit level of the ISIC classification (revision 3), but are originally at the 4-digit level in the data. Ownership based on whether or not female-owned capital is larger than 50% of total.

TABLE A3— EXIT PROBABILITY

| VARIABLES | (1)<br>Exit | (2)<br>Exit_no_2005 | (3)<br>Exit | (4)<br>Exit_no_2005 |
|---|---|---|---|---|
| female | 0.0582 | 0.0785 | 0.0698 | 0.0873 |
| | (0.0570) | (0.0600) | (0.0658) | (0.0705) |
| tfp | | | -0.0449** | -0.0384* |
| | | | (0.0191) | (0.0212) |
| age | | | -0.401*** | -0.407*** |
| | | | (0.0548) | (0.0596) |
| empl | | | -0.147*** | -0.143*** |
| | | | (0.0250) | (0.0269) |
| kl | | | -0.0319*** | -0.0364*** |
| | | | (0.0108) | (0.0120) |
| exporter | | | 0.0277 | -0.0333 |
| | | | (0.167) | (0.179) |
| importer | | | -0.0342 | -0.0342 |
| | | | (0.0581) | (0.0616) |
| Constant | -1.237*** | -1.255*** | 0.902 | -2.885*** |
| | (0.0213) | (0.0225) | (0.550) | (0.322) |
| Observations | 7,949 | 7,234 | 5,730 | 5,034 |
| Industry FE | NO | NO | YES | YES |
| Region FE | NO | NO | YES | YES |
| Year FE | NO | NO | YES | YES |

*Notes*: *** p<0.01, ** p<0.05, * p<0.1. Standard errors are clustered at the firm level. The dependent variable is a dummy variable taking value of 1 if the firm leaves the sample, and 0 otherwise. Columns (2) and (4) are run excluding 2005, since in that year a representative survey was run instead of the full census.

TABLE A4—EFFECTS OF FIRMS CHARACTERISTICS ON THE PRODUCTIVITY GAP, BY QUANTILE Q

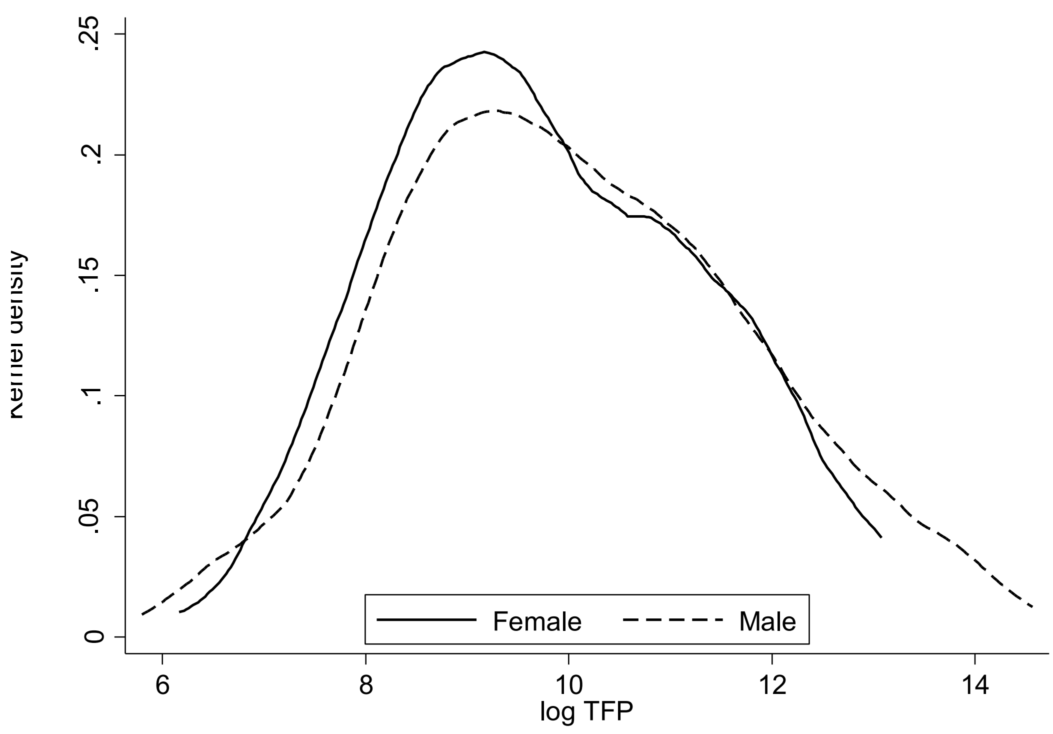
| Composition effects | q10 | q20 | q30 | q40 | q50 | q60 | q70 | q80 | q90 |
|---|---|---|---|---|---|---|---|---|---|
| age | 0.005 | 0.000 | -0.000 | -0.001 | -0.000 | -0.000 | -0.001 | -0.002 | -0.003 |
| | (0.007) | (0.003) | (0.001) | (0.001) | (0.001) | (0.001) | (0.002) | (0.004) | (0.005) |
| emp | -0.090*** | -0.091*** | 0.120*** | 0.136*** | 0.151*** | 0.203*** | 0.229*** | 0.251*** | 0.273*** |
| | (0.034) | (0.028) | (0.014) | (0.015) | (0.017) | (0.021) | (0.024) | (0.026) | (0.031) |
| kl | 0.003 | 0.016 | 0.018* | 0.032*** | 0.047*** | 0.058*** | 0.123*** | 0.097*** | 0.096*** |
| | (0.013) | (0.011) | (0.010) | (0.011) | (0.014) | (0.016) | (0.024) | (0.021) | (0.021) |
| exporter | 0.016* | 0.023** | -0.011*** | -0.008*** | -0.005 | -0.002 | 0.009 | 0.033*** | 0.100*** |
| | (0.009) | (0.011) | (0.003) | (0.003) | (0.004) | (0.005) | (0.006) | (0.009) | (0.019) |
| importer | -0.021 | -0.033*** | 0.019*** | 0.024*** | 0.027*** | 0.026*** | 0.015** | 0.013* | 0.018* |
| | (0.014) | (0.012) | (0.006) | (0.007) | (0.008) | (0.008) | (0.007) | (0.007) | (0.010) |
| total | 0.125* | 0.113** | -0.133*** | -0.093* | -0.016 | 0.097 | 0.219** | 0.365*** | 0.502*** |
| | (0.071) | (0.056) | (0.046) | (0.057) | (0.075) | (0.094) | (0.092) | (0.080) | (0.084) |
| Structural effects | | | | | | | | | |
| age | 0.942** | 0.048 | -0.516 | -0.734** | -1.020** | -0.746 | -0.735 | -0.591 | 0.467 |
| | (0.399) | (0.353) | (0.333) | (0.352) | (0.412) | (0.469) | (0.513) | (0.550) | (0.577) |
| emp | -0.234 | -0.249 | 0.203 | 0.060 | 0.011 | -0.000 | -0.099 | -0.637* | -0.324 |
| | (0.253) | (0.212) | (0.176) | (0.192) | (0.221) | (0.251) | (0.284) | (0.327) | (0.441) |
| kl | -0.044 | -0.279 | 0.352* | 0.479** | 0.316 | 0.697** | 1.594*** | 0.922** | 1.077** |
| | (0.218) | (0.187) | (0.180) | (0.207) | (0.231) | (0.278) | (0.364) | (0.380) | (0.513) |
| exporter | -0.011 | -0.027* | 0.005 | 0.003 | 0.006 | 0.005 | 0.011 | 0.023 | 0.007 |
| | (0.013) | (0.015) | (0.004) | (0.005) | (0.006) | (0.007) | (0.010) | (0.017) | (0.022) |
| importer | 0.027 | 0.115 | -0.121 | -0.044 | 0.006 | 0.060 | -0.015 | -0.004 | 0.209 |
| | (0.115) | (0.091) | (0.083) | (0.090) | (0.108) | (0.111) | (0.123) | (0.130) | (0.171) |
| total | -0.026 | -0.020 | 0.198*** | 0.255*** | 0.177** | 0.159* | 0.130 | 0.107 | 0.052 |
| | (0.091) | (0.065) | (0.060) | (0.063) | (0.072) | (0.083) | (0.091) | (0.093) | (0.110) |
| N | 7,546 | 7,546 | 7,546 | 7,546 | 7,546 | 7,546 | 7,546 | 7,546 | 7,546 |

*Notes*: *** p<0.01, ** p<0.05, * p<0.1. Results are based on the Firpo et al. (2007) quantile decomposition method described starting on page 14 . Coefficients reporting the contribution of Industry, Region and Year dummies to the differences in productivity are included but not reported for reasons of space.

TABLE A5—CHARACTERISTICS OF FIRMS AT THE BOTTOM/TOP OF THE PRODUCTIVITY DISTRIBUTION

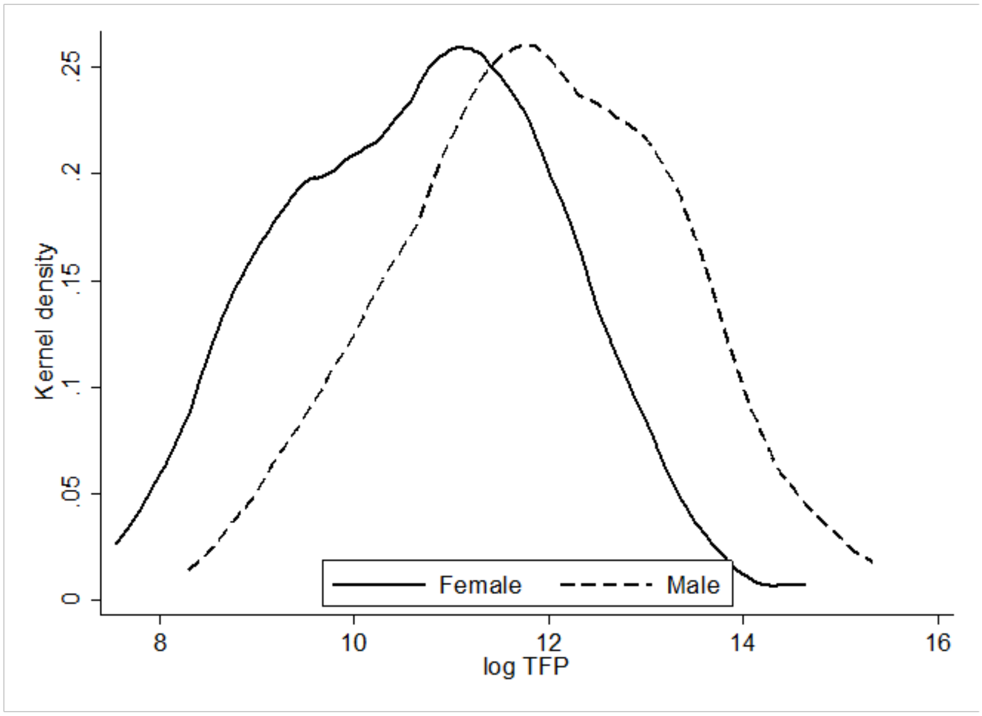
| | **Bottom 10%** | | | **Top 10%** | | |
|---|---|---|---|---|---|---|
| | **Female** | **Male** | **Fem/Male** | **Female** | **Male** | **Fem/Male** |
| N. of employees | 19.2 | 28.5 | 0.68 | 102.0 | 318.4 | 0.32 |
| Female/male workers | 1.1 | 0.5 | 2.23 | 0.4 | 0.5 | 0.70 |
| Female/male skilled workers | 0.7 | 0.4 | 1.61 | 0.6 | 0.4 | 1.39 |
| Female/male unskilled workers | 1.2 | 0.6 | 2.01 | 0.3 | 0.8 | 0.40 |
| wage_pc (birr) | 2937.9 | 4249.8 | 0.69 | 10367.8 | 9955.9 | 1.04 |
| wage_skilled (birr) | 2798.6 | 4149.6 | 0.67 | 12453.9 | 12613.4 | 0.99 |
| wage_female (birr) | 1899.0 | 2859.1 | 0.66 | 12033.6 | 11058.4 | 1.09 |
| wage_male (birr) | 2912.8 | 4278.0 | 0.68 | 12813.4 | 12883.0 | 0.99 |
| acces to credit (y/n) | 0.2 | 0.2 | 0.91 | 0.5 | 0.5 | 0.90 |
| fixed assets on empl (birr) | 29505.7 | 29601.6 | 1.00 | 96320.8 | 111581.4 | 0.86 |
| exporter (y/n) | 0.0 | 0.0 | .. | 0.1 | 0.3 | 0.30 |
| importer (y/n) | 0.5 | 0.6 | 0.79 | 0.6 | 0.6 | 0.97 |
| foreign (y/n) | 0.0 | 0.0 | .. | 0.0 | 0.1 | 0.30 |
| sales (birr) | 509578.3 | 886987.2 | 0.57 | 18900000 | 48600000 | 0.39 |
| value_added (birr) | 442123.5 | 658005.3 | 0.67 | 20300000 | 48400000 | 0.42 |
| First 3 industries (% on total) | 57.1% | 69.4% | 0.82 | 67.8% | 49.0% | 1.38 |
| based in Addis | 36.5% | 59.7% | 0.61 | 28.8% | 38.0% | 0.76 |
| N | 77 | 696 | 0.11 | 59 | 714 | 0.08 |

FIGURE A1. FEMALE- VS. MALE-OWNED FIRM PRODUCTIVITY DISTRIBUTIONS, ADDIS ABABA ONLY



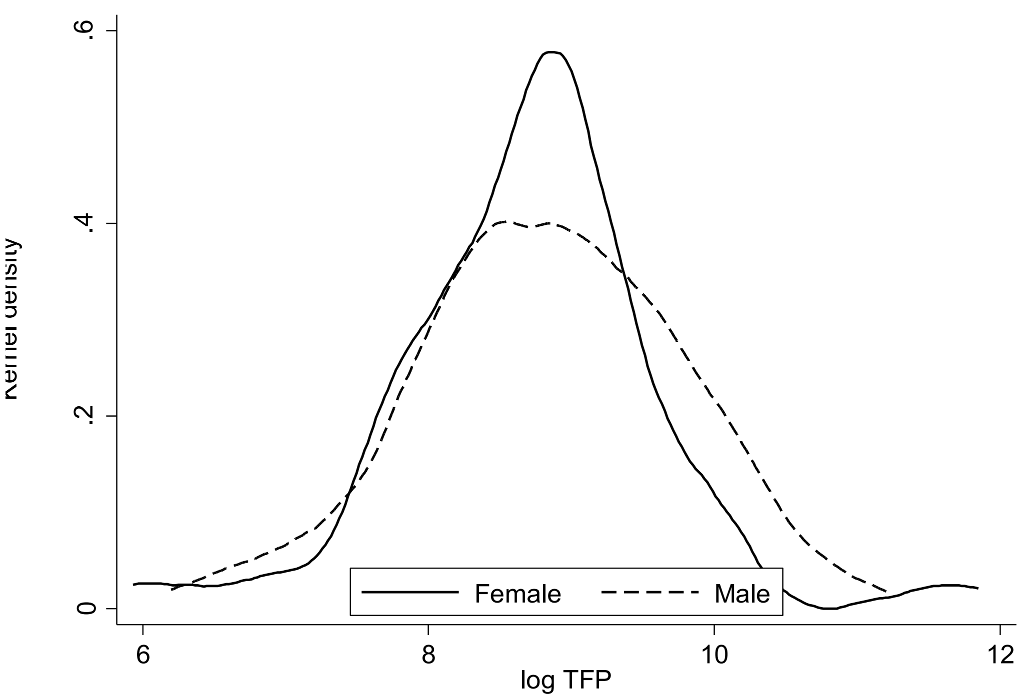

*Notes*: Densities of log TFP distributions are estimated using an Epanechnikov kernel function. Top and bottom 1% of log TFP distributions are trimmed (separately for male and female distributions).

FIGURE A2. FEMALE- VS. MALE-OWNED FIRM PRODUCTIVITY DISTRIBUTIONS, FOOD AND BEVERAGES (SITC 15) INDUSTRY ONLY



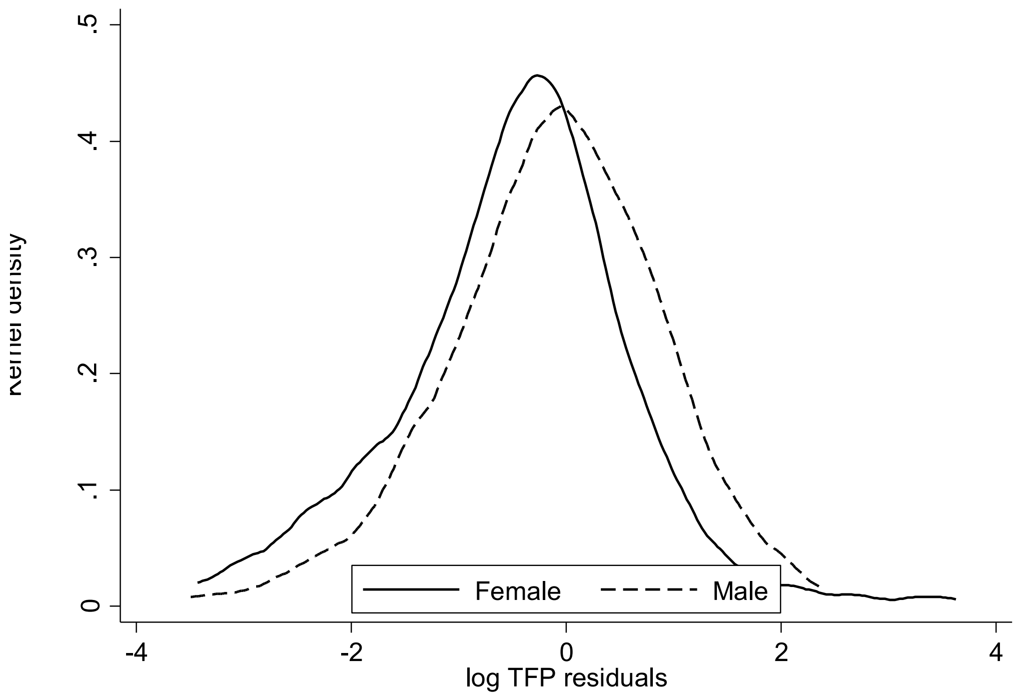

*Notes*: Densities of log TFP distributions are estimated using an Epanechnikov kernel function. Top and bottom 1% of log TFP distributions are trimmed (separately for male and female distributions).

FIGURE A3. FEMALE- VS. MALE-OWNED FIRM PRODUCTIVITY DISTRIBUTIONS, MINERALS (SITC 26) INDUSTRY ONLY



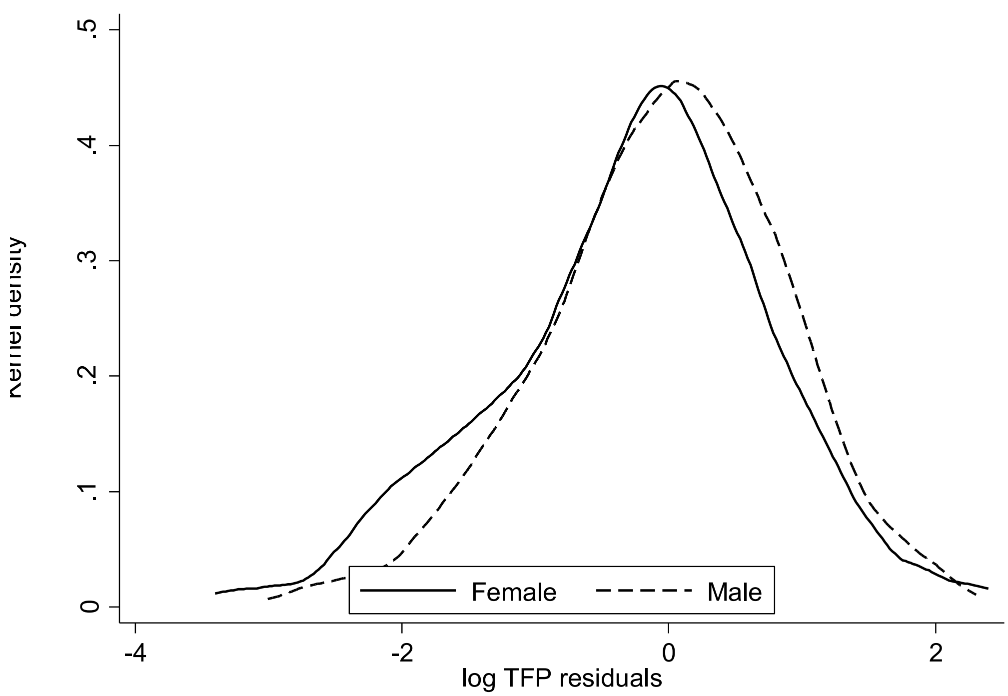

*Notes*: Densities of log TFP distributions are estimated using an Epanechnikov kernel function. Top and bottom 1% of log TFP distributions are trimmed (separately for male and female distributions).

FIGURE A4. FEMALE- VS. MALE-OWNED FIRM PRODUCTIVITY RESIDUAL DISTRIBUTIONS, CONTROLLING FOR SECTORS




*Notes*: Residuals are obtained from a regression of log TFP on industry dummies. Densities are estimated using an Epanechnikov kernel function. Top and bottom 1% of log TFP residual distributions are trimmed (separately for male and female distributions).

FIGURE A5. FEMALE- VS. MALE-OWNED FIRM PRODUCTIVITY RESIDUAL DISTRIBUTIONS, CONTROLLING FOR SECTORS, FIRM SIZE AND CAPITAL INTENSITY




*Notes*: Residuals are obtained from a regression of log TFP on industry dummies, log of number of employees, and log of ratio of capital to the number of employees. Densities are estimated using an Epanechnikov kernel function. Top and bottom 1% of log TFP residual distributions are trimmed (separately for male and female distributions).

## Appendix B: Description of the Combes et al. (2012) methodology

To better understand the gist of Combes et al.'s (2012) approach, let us consider two cumulative distributions, $F_f$ for female-owned firms and $F_m$ for male-owned firms, with some common underlying distribution $F$. Further assume that $F_f$ ($F_m$) can be obtained by shifting $F$ rightward by a constant $A_f$ ($A_m$), dilating $F$ by a factor $D_f$ ($D_m$), and left-truncating a share $S_f$ ($S_m$) ϵ [0, 1[ of F; or mathematically:

$$F_f(\phi) = max\left\{0, \frac{F\left(\frac{\phi - A_f}{D_f}\right) - S_f}{1 - S_f}\right\}$$

and

$$F_m(\phi) = max\left\{0, \frac{F\left(\frac{\phi - A_m}{D_m}\right) - S_m}{1 - S_m}\right\}$$

where $\phi$ stands for productivity or, more specifically, log TFP. Moreover, if we define $D \equiv \frac{D_f}{D_m}$, $A \equiv A_f - DA_m$, and $S \equiv \frac{S_f - S_m}{1 - S_m}$,
then it is relatively straightforward to show that the following relationship between $F_f$ and $F_m$ holds[18]:

$$F_f(\phi) = max\left\{0, \frac{F_m\left(\frac{\phi - A}{D}\right) - S}{1 - S}\right\} \; if \; S_f > S_m$$

$$F_m(\phi) = max\left\{0, \frac{F_f(D\phi + A) - \frac{-S}{1 - S}}{1 - \frac{-S}{1 - S}}\right\} \; if \; S_f < S_m$$

This relation indicates that one distribution, say $F_f$, can be obtained as a transformation of the other, say $F_m$, without having to specify the common underlying distribution $F$. Combes et al. (2012) translate the above expressions in quantile functions that can be estimated to derive parameters $A$, $D$, and $S$, i.e., respectively, the relative shift, relative dilation, and relative left truncation that need to be applied to $F_m$ to approximate $F_f$.

[18] The simple proof of this relation can be found in Appendix C of Combes et al. 2012, 2589-90).